A stochastic control approach to robust duality in utility maximization

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Abstract

A celebrated financial application of convex duality theory gives an explicit relation between the following two quantities: (i) The optimal terminal wealth $X^*(T) := X_{\varphi^*}(T)$ of the problem to maximize the expected U-utility of the terminal wealth $X_\varphi(T)$ generated by admissible portfolios $\varphi(t); 0 \le t \le T$ in a market with the risky asset price process modeled as a semimartingale; (ii) The optimal scenario $\frac{dQ^*}{dP}$ of the dual problem to minimize the expected V-value of $\frac{dQ}{dP}$ over a family of equivalent local martingale measures Q, where V is the convex conjugate function of the concave function U. In this paper we consider markets modeled by Itô-Lévy processes. In the first part we extend the above result in this setting, based on the maximum principle in stochastic control theory. We prove in particular that the optimal adjoint process for the robust primal problem coincides with the optimal density process, and that the optimal adjoint process for the robust dual problem coincides with the optimal wealth process. We get moreover an explicit relation between the optimal portfolio φ^* and the optimal measure Q^*. We also obtain that the existence of an optimal scenario is equivalent to the replicability of a related T-claim. In the second part we present robust (model uncertainty) versions of the optimization problems in (i) and (ii), and we prove a relation between them. In particular, we show explicitly how to get from the solution of one of the problems to the other. We illustrate the results with explicit examples.

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